Beta function at BPM's location

Xiaobiao Huang

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- The idea is from Shekhar Shukla's report.

1 Principle

An ORM matrix element is

$$M_{ij} = \frac{y_i}{\theta_j} = \frac{\sqrt{\beta(s_i)\beta(s_j)}}{2\sin(\pi\nu)}\cos(\pi\nu - |\psi(s_i) - \psi(s_j)|), \qquad (1)$$

where $\beta(s_i)$,$\psi(s_i)$ are at i'th BPM, $\beta(s_i)$,$\psi(s_i)$ are at j'th kicker.

For booster, i'th BPM and i'th kicker are practically located at the same place. This leads to

(1)

$$M_{ii} = \frac{y_i}{\theta_i} = \frac{\beta(s_i)}{2\sin(\pi\nu)}\cos(\pi\nu) \qquad (2)$$

(2)

$$M_{ij} = M_{ji} \qquad (3)$$

2 A description of the raw data

For each M_{ij}, there are 8-10 measurements with bump current ranging from -1.0A to 1.0A. A linear fitting is used to get y_i/I_j and then it is converted to M_{ij} according to

$$\theta = \frac{I * 3000 \times 10^{-6} Tm}{3.3357 p[GeV/c] 5.6A} \qquad (4)$$

Criterion for an accepatable fitting: $|\sigma_b/b| < 0.1$. Other elements are set to zero.

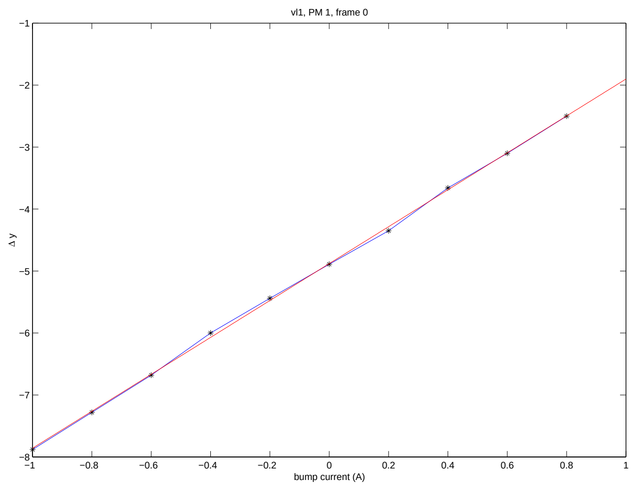

Figure 1: y = -4.880182 + (2.978182)*x; σ_a = 0.012733; σ_b = 0.021837, χ^2 = 0.012589;

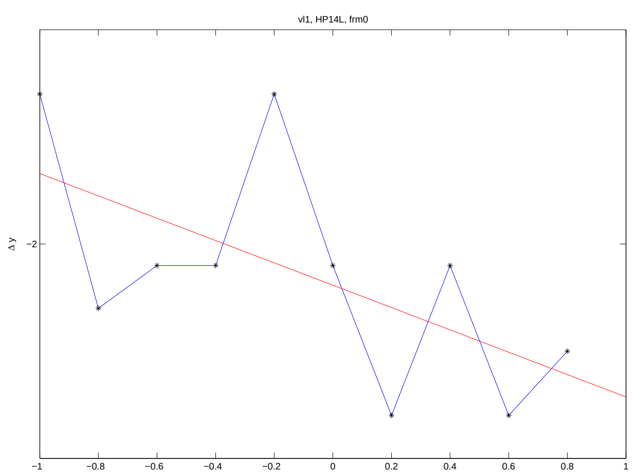

Figure 2: y = -2.019212 + (-0.052121)*x; σ_a = 0.014117; σ_b = 0.024211; χ^2 = 0.015475

frm	time(ms)	momentum(GeV/c)	number of zero element
0	2.0	1.0246	240
1	3.8	1.2021	258
2	5.5	1.4681	246
3	7.0	1.7937	291
4	8.5	2.1715	359
7	12.70		589
10	16.64		779
13	20.52		1270

3 Fitting scale factors: BPM gain and kicker calibration

1. by comparing model and experiment data
$$\chi^2 = \sum(M_{model,ij} - \frac{M_{data,ij}}{b_i k_j})^2 \tag{5}$$

frame	0	1	2	3	4
χ_i^2	1.1e5	2.9e4	3.3e4	5.3e4	6.6e4
χ_f^2	2.8e4	5.1e3	2.4e3	1.8e3	9.5e2
N	2066	2046	2058	2013	1945

Table 1: χ^2 as defined in 5;

2. by experiment data and using the fact M_{ij}=M_{ji} for Booster,i.e.
$$\frac{M_{data,ij}}{b_i k_j} = \frac{M_{data,ji}}{b_j k_i} \tag{6}$$
So we can adjust b_i and k_j to minimize
$$\chi^2 = \sum(\frac{M_{data,ij} b_j k_i}{b_i k_j} - M_{data,ji})^2 \tag{7}$$
This will enable us to get $\frac{b_i}{k_i}$ with experiment data only. We can then determine b_i and k_j without ambiguity and do a cross check.

3. Result of b_i and k_j

frame	0	1	2	3	4
χ_i^2	14514	12759	13196	13815	10901
χ_f^2	3472	126	119	92	70
N	913	951	955	922	884

Table 2: χ^2 as defined in (7);

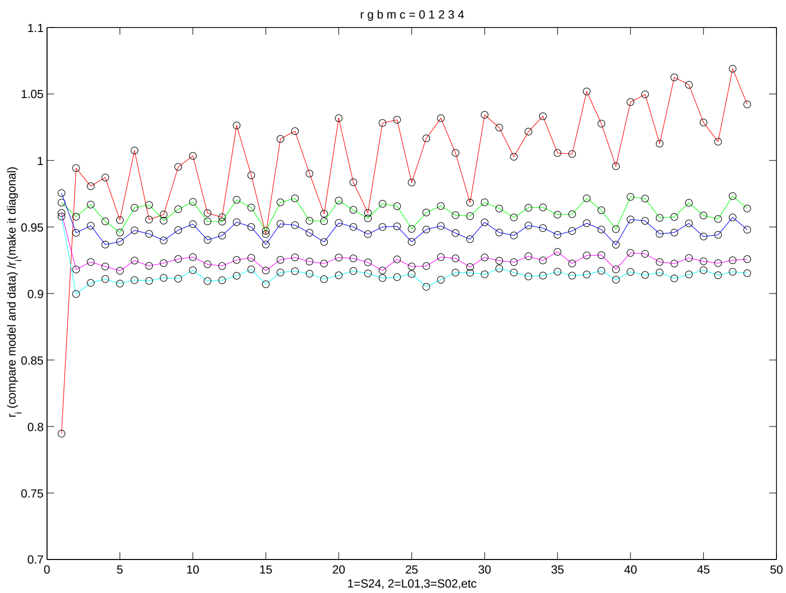

Figure 3: Compare the ratio of b_i/k_i obtained by the two methods; Is the comparsion trivial?

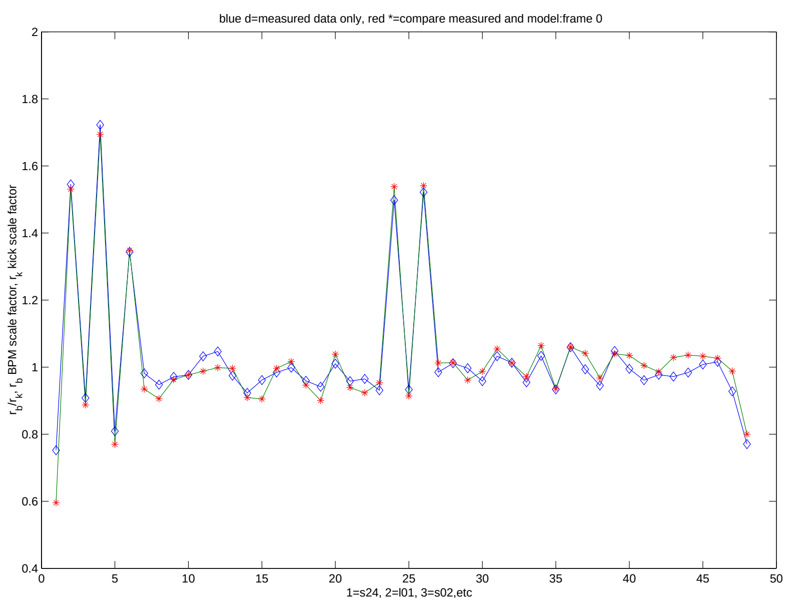

Figure 4: b_i/k_i obtained by the two methods; frame 0;

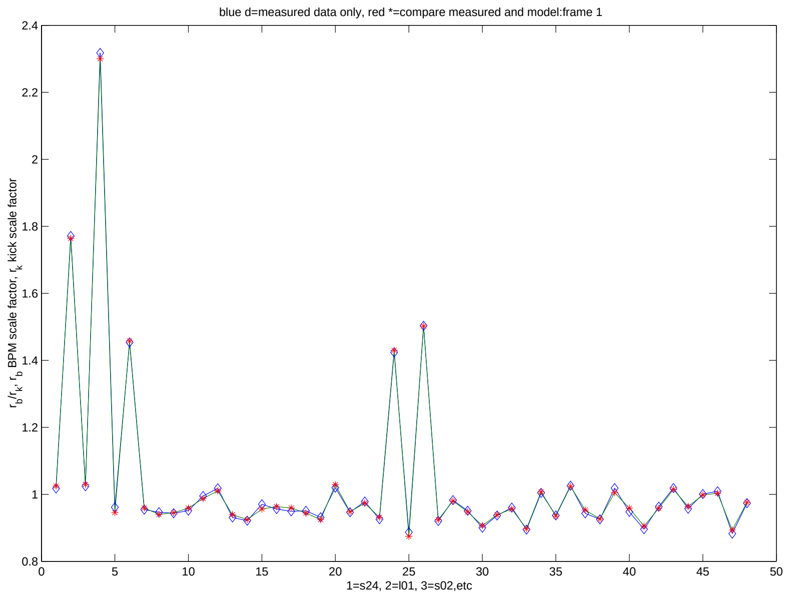

Figure 5: b_i/k_i obtained by the two methods; frame 1;

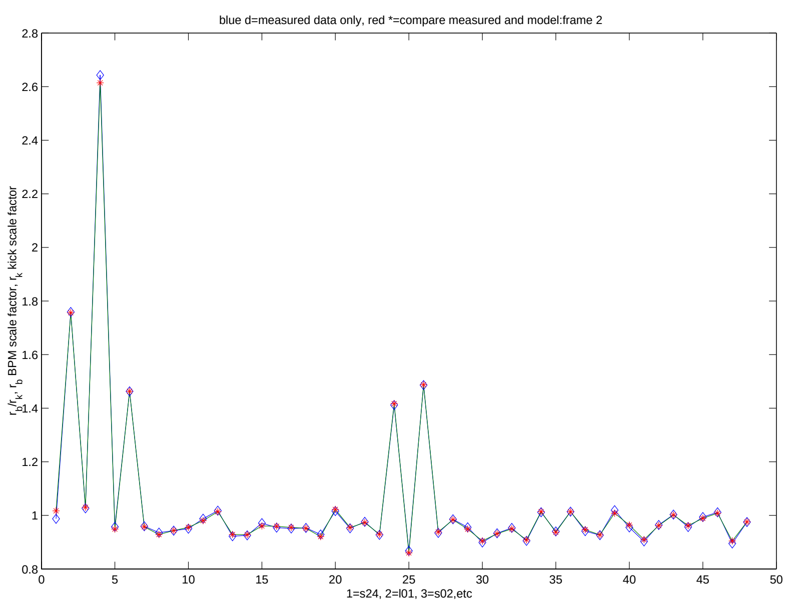

Figure 6: b_i/k_i obtained by the two methods; frame 2;

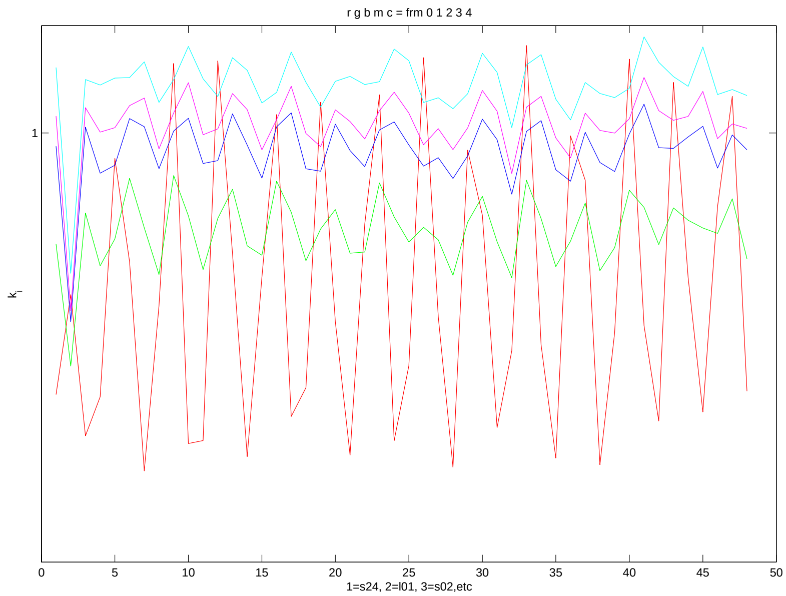

Figure 7: scale factors for kicker calibration; The averages for frame 0 to 4 are 0.8549, 0.9081 , 0.9840, 1.0082, 1.0449, 1.0569;

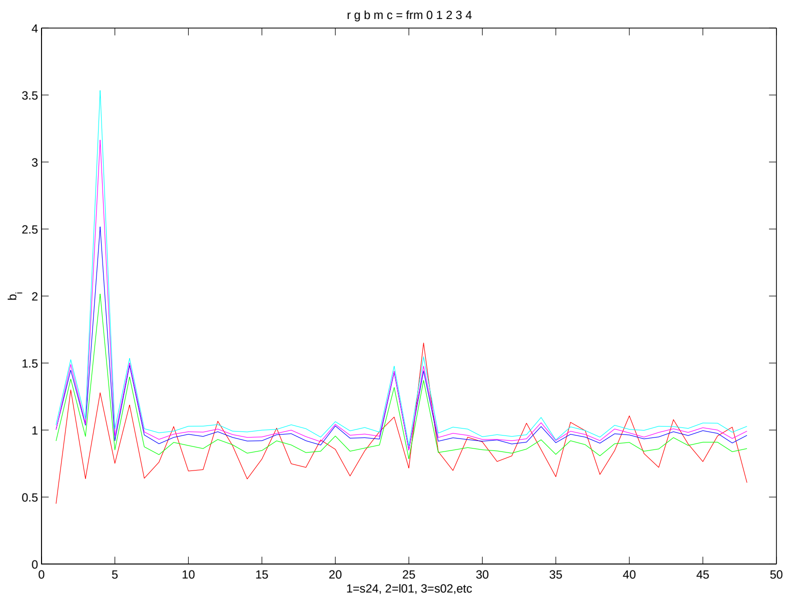

Figure 8: scale factors for BPM gain;

4 Compare M_{ii}, i.e., β_y at BPM's location

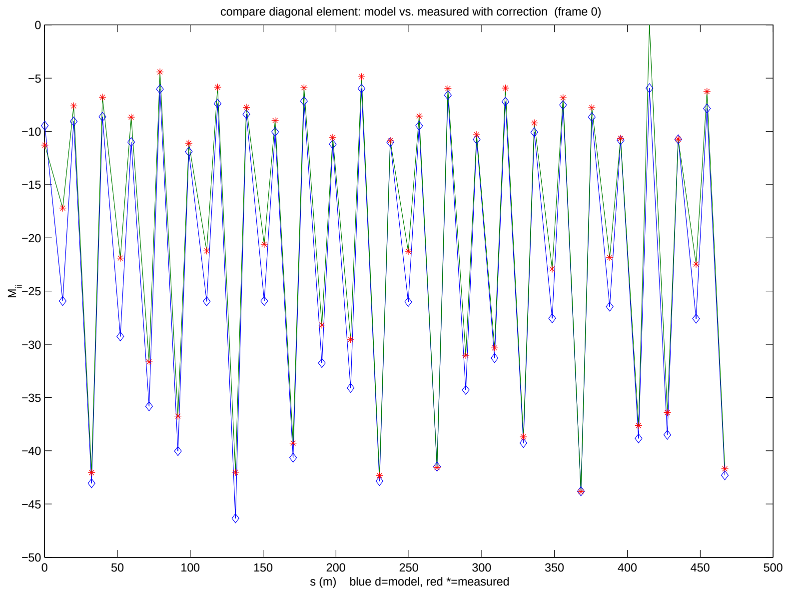

Figure 9: Diagonal elements of M matrix: model vs. measured; Frame 0;

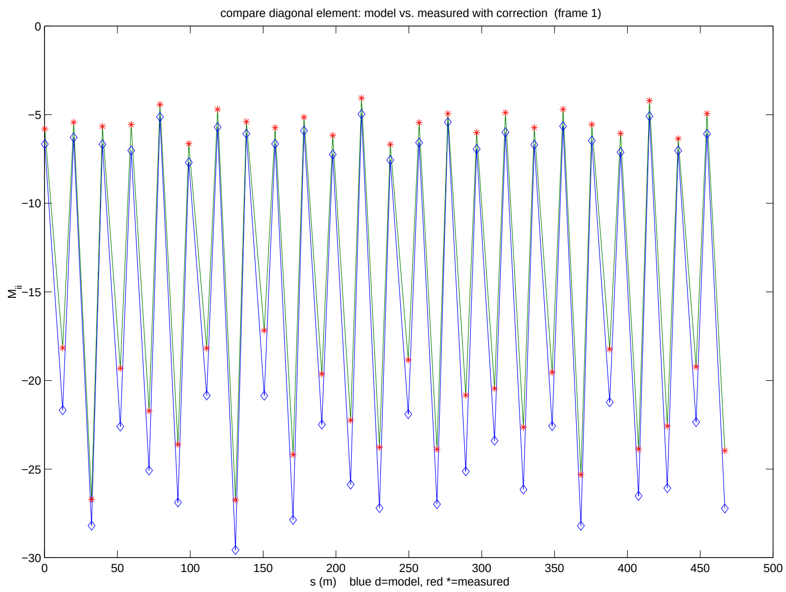

Figure 10: Diagonal elements of M matrix: model vs. measured; Frame 1;

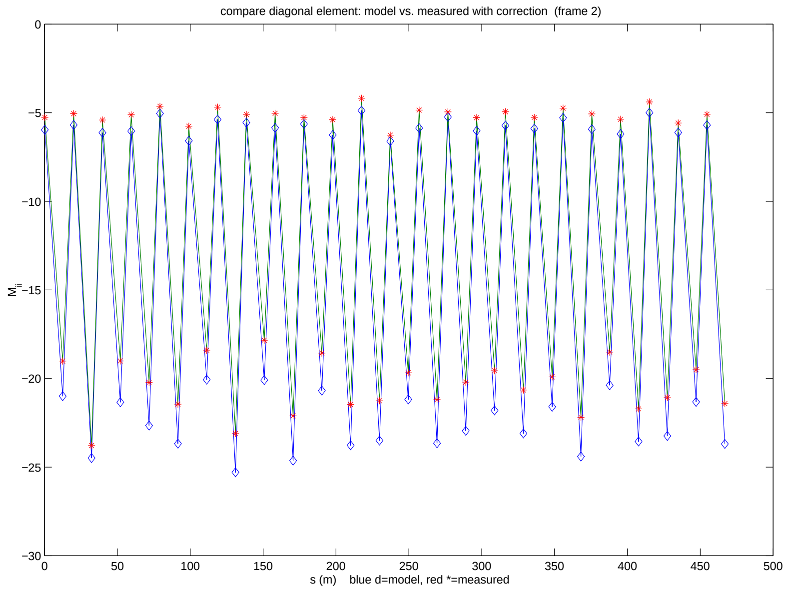

Figure 11: Diagonal elements of M matrix: model vs. measured; Frame 2;

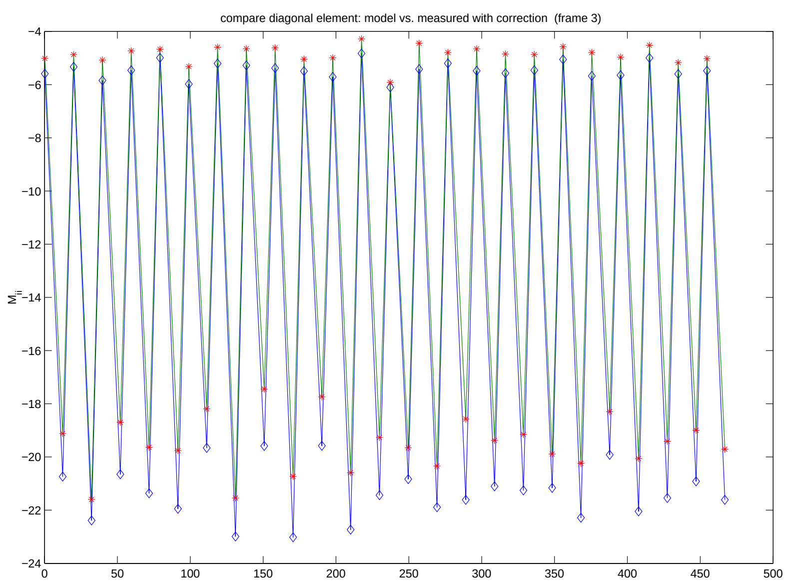

Figure 12: Diagonal elements of M matrix: model vs. measured; Frame 3;

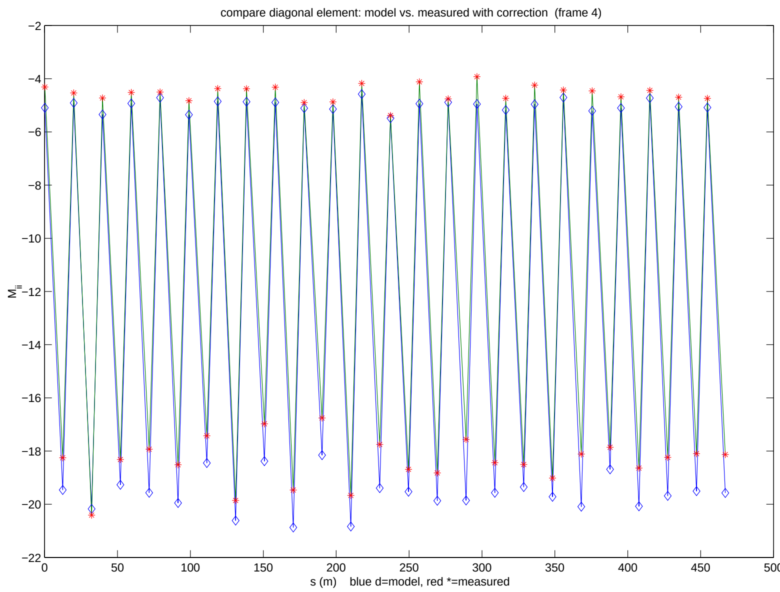

Figure 13: Diagonal elements of M matrix: model vs. measured; Frame 4;

frame	0	1	2	3	4	5
average ratio	0.8938	0.8616	0.8963	0.9020	0.9229	0.9527
$\tan(\pi\nu^{model})$	-0.2911	-0.3876	-0.4200	-0.4407	-0.5109	-0.5606
$\tan(\pi\nu^{measured})$	-0.3234	-0.4425	-0.4622	-0.4818	-0.5074	-0.5318
model tune	6.90735	6.87662	6.86632	6.85973	6.84910	6.83737
'measured' tune	6.8971	6.8591	6.8529	6.8467	6.8385	6.8307

Table 3: The measured tune is $(atan(tan(\nu^{model})/r)/\pi) + 7.0$

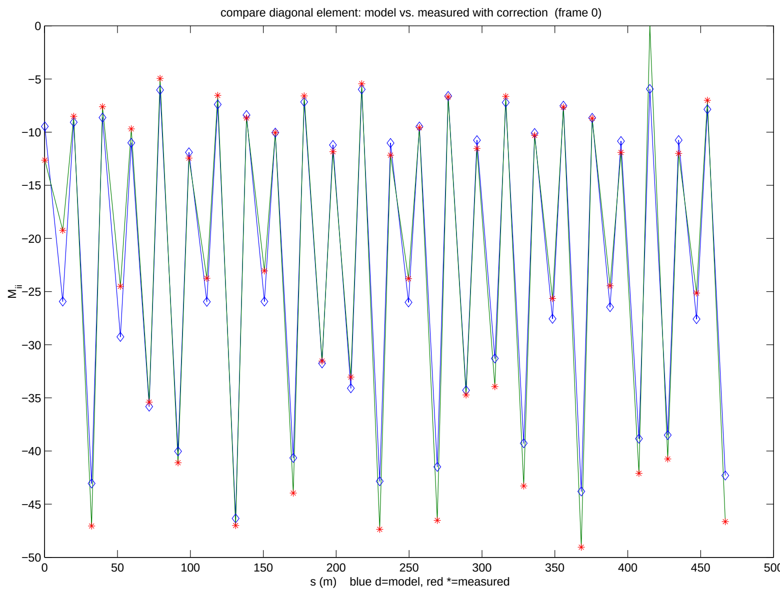

Figure 14: Diagonal elements of M matrix: model vs. measured; Frame 0;

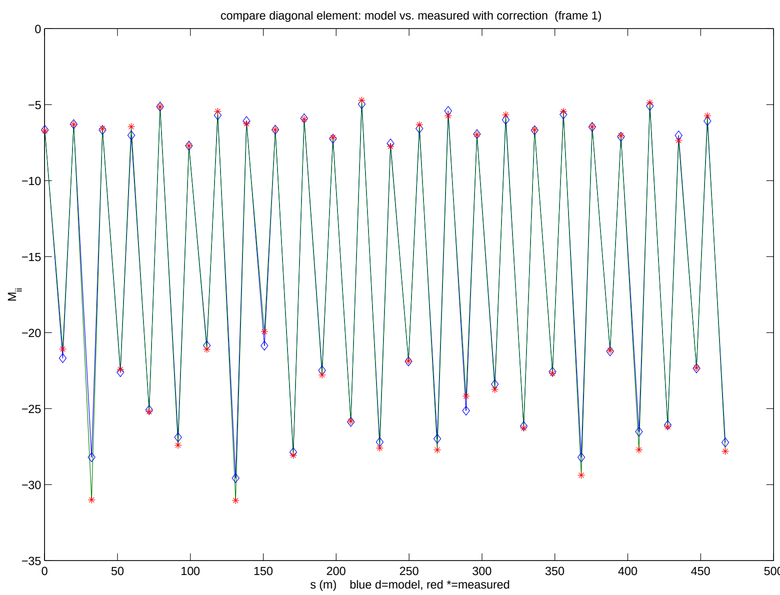

Figure 15: Diagonal elements of M matrix: model vs. measured; Frame 1;

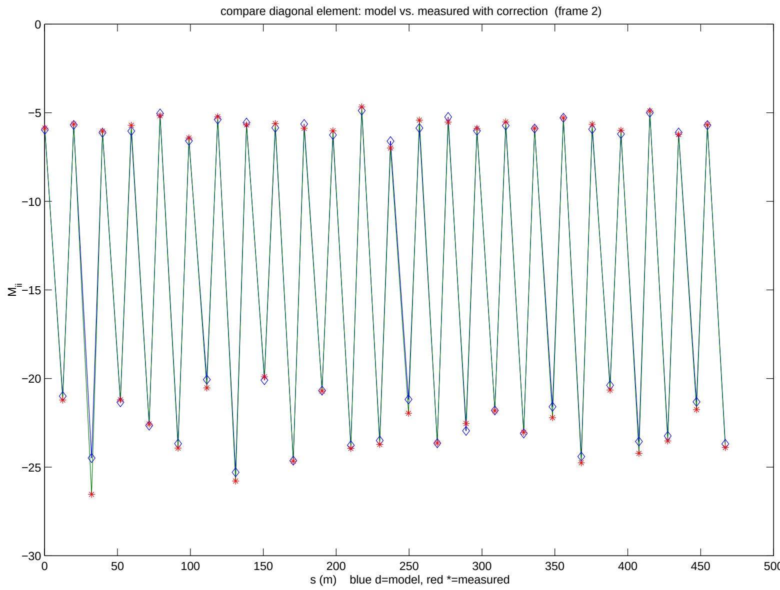

Figure 16: Diagonal elements of M matrix: model vs. measured; Frame 2;

Student Version of MATLAB

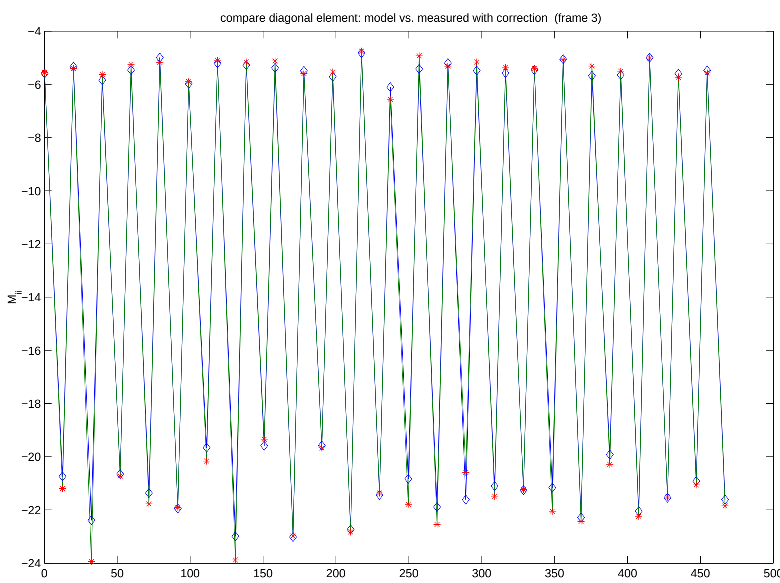

Figure 17: Diagonal elements of M matrix: model vs. measured; Frame 3;

Student Version of MATLAB

5 summary

1. Frame 0 (at 2ms) is different. Because model is not accurate for this one ?
2. Scale factors from the two methods agree to each other by showing the same pattern
3. Scale factors can be determined without ambiguity.
4. β_y from model agrees to $beta_y$ from measurement except for frame 0
5. We can even get the tunes

BUT

1. Does the two fitting methods really confirm each other?
2. Can we trust the 'measured' tunes and beta functions?

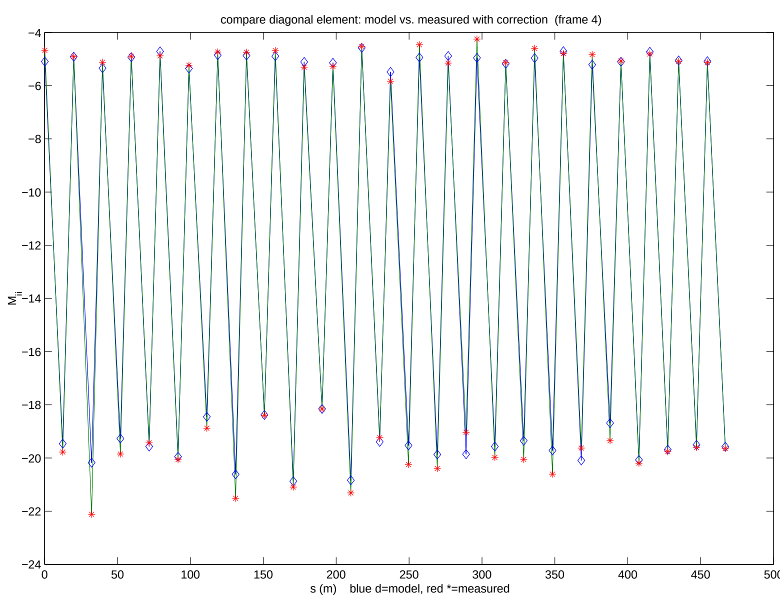

Figure 18: Diagonal elements of M matrix: model vs. measured; Frame 4;